Considering Mathematics Instruction

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INSPIRING LEARNERS
Student Achievement
PLAN
Engagement
Diversity & Inclusion
ACT
ASSESS
Forward Thinking
REFLECT
Wellness
Collaborative
Caring and Responsive
Positive Character
Differentiates Instruction
Communicator
Promotes Student Voice and Choice
Contributing Citizen
Reflective Practitioner
Creative Thinker
Utilizes Universal Design
Critical Thinker
Vision of Educator
Vision of Student
Overall Goal
Optimizing Learning & Well-Being for All Students to Achieve Excellence
2015-2016 Goal
Every Student will improve their ability to problem solve and communicate their thinking.
System Expectation
Students and educators will engage in inquiry through an ongoing collaborative cycle of learning (Plan, Act, Assess, Reflect)
Educator and Student Learning Focus
Instructional Leadership
Assessment for Learning - Descriptive Feedback
Choice of High-Yield Instructional Tools
Small Group Instruction
Open-Ended Critical Questions
2015-16 Target Areas
Collaboration
Mathematics
Values
High Expectations
Continuous Improvement
Hope and Optimism
Respectful Relationships
Safe, Inclusive, Equitable Learning Community
Championing Public Education
TOMORROW'S LEADERS

Agenda

- Backwards design to plan instruction (more focus on1P and 2P)
- Differentiating instruction through open questions and parallel tasks

What do you think?

Is it good enough if students can answer A and not B?

A: What is x if 3/x = 14/30?

B: How do you know that x has to be more than 6 without actually solving?

It would be great

- To focus our teaching more on ideas that underlie the math curriculum than performances

Bigger ideas are important

- To make the curriculum manageable
- To make the curriculum more meaningful

We want

- Questions and tasks that allow for richer discussion and not just a quick check on performance

This is true

- No matter what the topic

For example

- Proportional thinking

While we still want

- Kids to be able to solve proportions…

We also want

- Students to have a sense that the two sides of a proportion describe equivalent ratios so their “sizes” are equal.
- Students to be able to estimate solutions to proportions.

We also want

- Students to use the equality of ratios to use efficient strategies to solve proportions.
- Students to realize why we can multiply both terms of a ratio by the same amount, but not add the same amount to both terms without disturbing the ratio.

We also want

- Students to recognize that visual tools can be useful to estimate solutions to proportions.

So I might ask

- A ratio is equivalent to 4:10.
- Can its terms be 40 apart?
- Can they be 36 apart?

So I might ask

- 5/36 = 27/x
- Without setting up any equations, suggest a good estimate and say why it's good.

So I might ask

- Why might you solve these two proportions using different strategies?

12/39 = 4/x 14/42 = 8/x

So I might ask

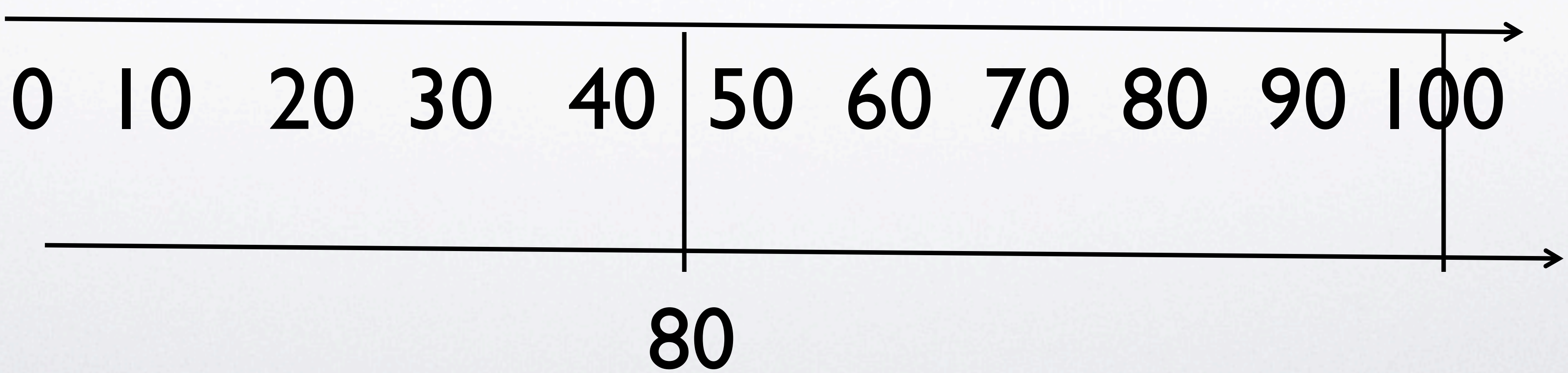
- Without solving, tell which of these two proportions have the same solution and why.

14/22 = x/39 OR 7/11 = x/39 OR

15/23 = x/39

So I might ask

- How might this picture help you estimate the solution to the proportion 45/100 = 80/x?



For example

- Quadratics

While we still want

- Representing and interpreting quadratics in various forms
- Determining max/min values
- Factoring quadratics

WE ALSO NEED

Quadratics

- Different forms of a quadratic are useful for different reasons
- How quadratics differ from other relationships

Quadratics

- Recognizing why quadratics might have maxima or minima, but not both
- Associating quadratics with “projectile” motion, but also certain measurements, typically area

Quadratics

- How factoring polynomials is like factoring numbers
- How factoring polynomials is about forming rectangles

Quadratics

- An ability to predict the relationship between the parameters in a quadratic expression and the effects on a graph

So I might ask

- You have to describe a quadratic relationship.
- When might you rather have the table of values? the graph? The equation?
- Factored form? standard form?

So I might ask

- Describe four ways that make a quadratic different from a linear relationship.

So I might ask

- Create a situation involving a quadratic relationship where the maximum value is 22 when the independent variable has a value of 4.
- Tell one other fact about the situation.

So I might ask

- Describe a situation that is quadratic that has a maximum.
- Why is there no minimum?

So I might ask

- Describe three quadratic relationships involving measurements that are quadratic. Tell why.
- Then describe two that are not quadratic and tell why.

So I might ask

- How is factoring $x^2 - 5x + 6$ like factoring numbers? How is it different?

So I might ask

- How might this table help you factor $x^2 + 2x + 1$?

So I might ask

x	$x^2 + 2x + 1$
0	1
1	4
2	9
3	16

So I might ask

- How can building a rectangle of tiles help you factor 56?
- How can building a rectangle of algebra tiles help you factor $x^2 + 5x + 6$?

So I might ask

- Choose a quadratic, but not $y = x^2$, that you would find easy to predict the graph of. Draw to check. Why was it easy? OR
- Predict how the graph of $y = 2(2x – 2)^2 + 2$ is different from the graph of $y = \frac{1}{2}(x/2 – \frac{1}{2})^2 + \frac{1}{2}$?

Consider measurement

I want students to realize

- That formulas are a way to make difficult to take measurements easier to calculate
- That the variables in a formula tell me which measurements are related to which and how

I want students to realize

- That the units I use play a big role in the sizes I use to describe measurements
- That some measures are related to others and so alternative formulas are possible
- That if some measures of two figures are related, some, but may not all, measures of those figures are related

So I might ask

- Why would it be really hard to figure out the volume of a cylinder if you didn't know the formula (or would it)?

So I might ask

- I am figuring out the volume of a cone.
- What measurements do I need to know? Which don’t I care about?
- Are there choices?

So I might ask

- I know $\text{Volume}_{\text{cone}} = 1/3\ \pi\ r^2 h$.
- $\text{Volume}_{\text{sphere}} = 4/3\ \pi r^3$.
- How many measurements of the cone do I need to figure out its volume? The sphere?
- Why does that make sense?

So I might ask

- How could the volume of a sphere be both 7.8 cubic units and 7800 cubic units?
- Can every measurement be made to sound bigger? Smaller?

So I might ask

- One formula for the volume of a cone is $V= 1/3\pi r^2h$.
- What are some alternate formulas that don't involve the radius? Why are they also legitimate?

So I might ask

- There are two cones.
- One has twice the height of the other.
- Do you know how their volumes compare?
- What if the one with the double height has half the radius?

Algebraic equations/ expressions

- I want students to know

that

- Any equation or expression can describe many situations
- Any situation that can be modelled algebraically can be modelled in multiple ways.
- Algebra is often used to describe numerical generalizations.

So I might ask

- Describe three different situations that the expression $3x + 2y – 8$ could describe.

Perhaps

- I filled 3 boxes with the same number of chocolates and 2 other boxes with the same number of chocolates and then ate 8 of them. How many chocolates are left?

So I might ask

- How can you describe the situation below algebraically two different ways?
- The perimeter of a rectangle whose length is three times the width is 58.

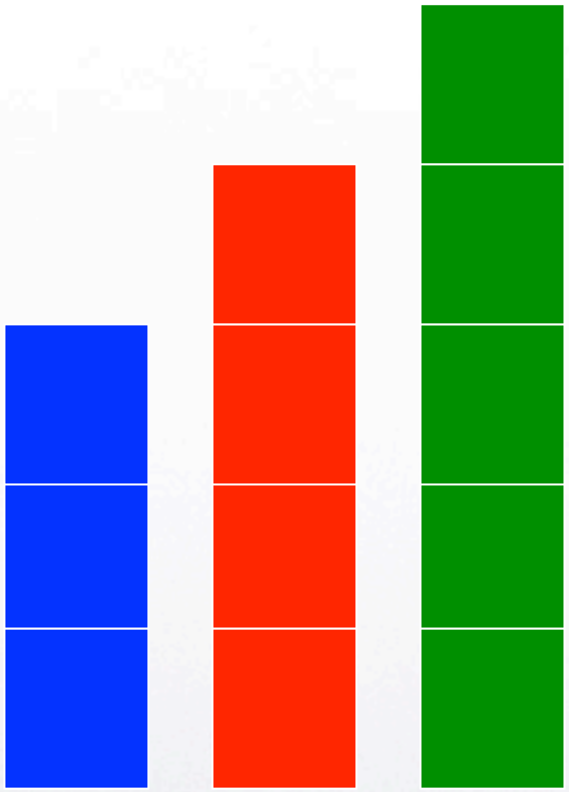
Or I might ask

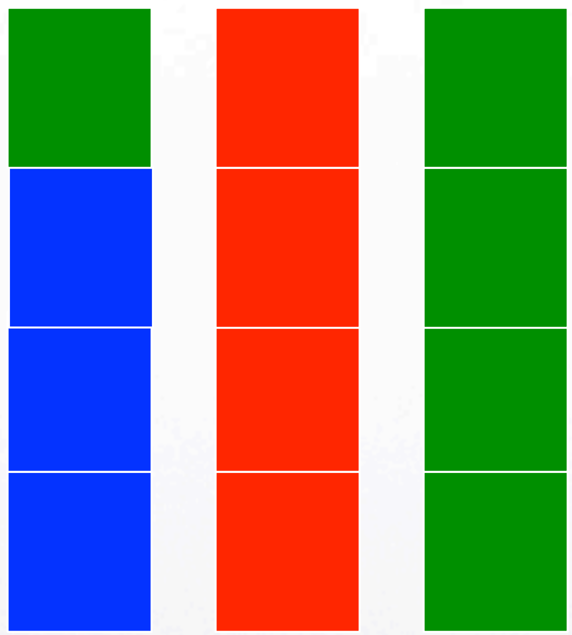
- Choose three consecutive whole numbers. Add them.
- Repeat.
- What do you notice?
- How could you describe this algebraically?

For example

- $5 + 6 + 7 = 18$
- $20 + 21 + 22 = 63$
- $49 + 50 + 51 = 150$

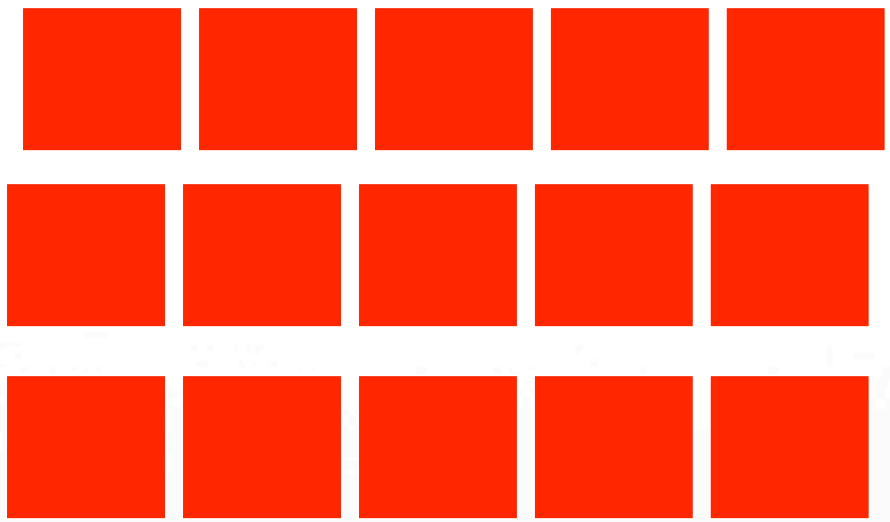
- $n + (n+1) + (n+2) = 3(n+1)$ OR
- $(n - 1) + n + (n + 1) = 3n$

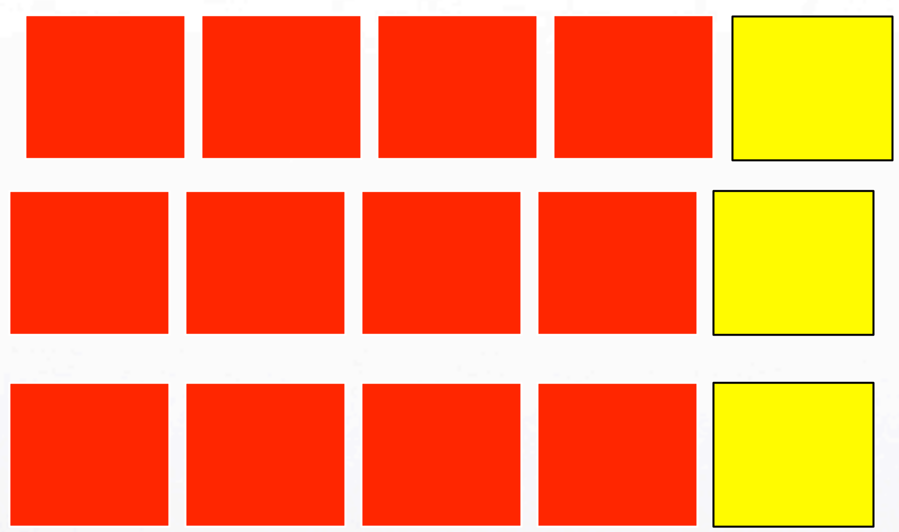


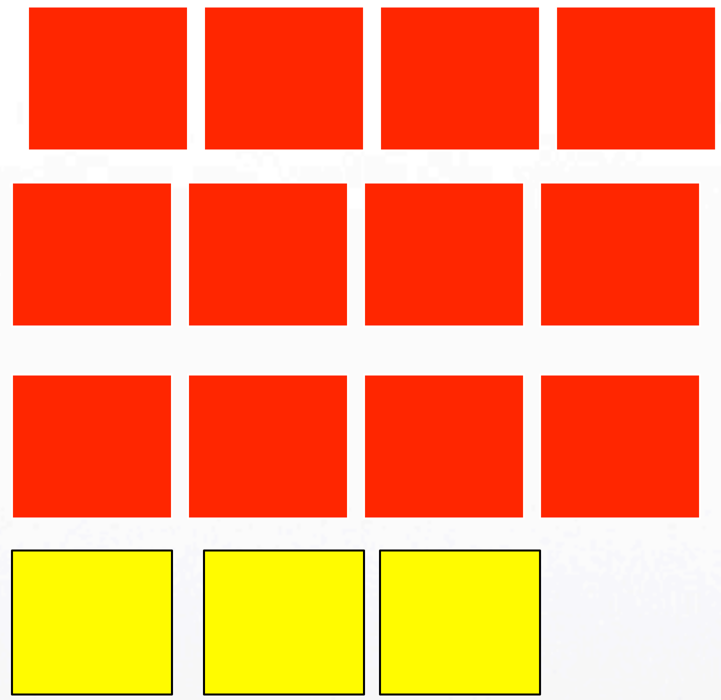


Or

- Multiply a whole number by the number two greater.
- Square the number between those two values.
- What do you notice?
- How can you say this algebraically?







For example

- $5 \times 7 = 6^2 - 1$
- $8 \times 10 = 9^2 - 1$
- $99 \times 101 = 100^2 - 1$
- $n(n + 2) = (n + 1)^2 - 1$ OR
- $(n-1)(n+1) = n^2 - 1$

Once you decide

- what is important, that should be the focus of both the kind of instructional situations and the kind of assessment questions you ask.

Your turn

- Choose another topic you teach.
- What are the important IDEAS (not skills) you want kids to be able to make sense of.
- Create a question for a few of them that gets there.

You probably noticed

- The key, for me, is posing good questions.

Maybe

- There are some questions that require repeating what you've shown them.

Some of

- The straightforward is okay, but it can’t be the majority of what you do.
- Otherwise students are not really learning anything new.

Knowledge vs understanding

- I could ask:
- What is the slope of the line that goes through (3,2) and (4, 5)?

OR

Knowledge vs understanding

- Two lines go through (3,2).
- One has a greater slope than the other.
- How do the intercepts relate?

Knowledge vs understanding

- A batch of cookies needs 2 ¼ cups of flour for every ¾ cup sugar.
- How much flour would you need if you only use ½ cup sugar?

OR

Knowledge vs understanding

- A batch of cookies uses 2 ¼ cups of flour for every ¾ cup sugar.
- You have a measuring cup with no markings on it. You fill it with sugar. Can you be sure how much flour to pour in? Explain.

Knowledge vs understanding

- Solve $100x + 6 = 87x + 2$

OR

Knowledge vs understanding

- WITHOUT SOLVING, tell why the solution to $100x + 6 = 87x + 2$ HAS TO be negative.

Knowledge vs understanding

- WITHOUT SOLVING, tell why the solution to $100x + 6 = 87x + 2$ HAS TO be negative.

Knowledge vs understanding

- What is $(3x + 2) - (2x^2 - 8)$?

OR

Knowledge vs understanding

- When you subtract two binomials, show that your answer could be a monomial, a binomial, or a trinomial.

Knowledge vs understanding

- Where do the lines $y = 3x + 5$ and $y = 2x – 8$ intersect?

OR

Knowledge vs understanding

- How can you predict that the lines $y = 3x + 5$ and $y = 2x - 8$ intersect when x is negative without actually figuring out where?

You try

- Start with a knowledge question you typically ask.
- Try to change it into an understanding question.

Thinking questions

- These may be understanding questions or may be “thinking” questions.
- Either way, the student needs to think.

Here are some examples

- A cylinder and a prism have the same volume.
- Choose a volume and choose dimensions to make it happen.

Here are some examples

- A rectangle has a length two times its width.
- Another rectangle with the same perimeter has a length five times its width.
- Can you be sure which has more area without knowing the dimensions? Explain.

Here are some examples

- A shape made up of two trapezoids, two triangles and a parallelogram has an area of 50 cm^2. What might all the dimensions be?

Here are some examples

- A right triangle is on a coordinate grid.
- One side is on the line $y = -2$.
- On what lines might the other sides be?

Here are some examples

- A line goes through Quadrants II, III and IV.
- Tell everything you know about the line.

Here are some examples

- You bought a number of $5 items and a number of $2 items and spent $80.
- Your friend did the same.
- Could you have bought one more $5 item than her?
- Could you have bought 6 more $5 items?

Here are some examples

- You saved 40% on a jacket and ended up paying the same amount as you did for a pair of shoes that was 20% off.
- How did the original prices compare?

For example

- Which equation does not belong? Why?

$3x - 4 = 2x - 7$

$6/x = -2$

$2x = -8$

$5x + 8 = -7$

For example

- If you know 20% of an amount, what other percents of it would be quick for you to figure out?
- Would be possible to figure out, but not quick?

For example

- You know that a line goes through the point $(1, -1)$ and slants down to the right.

- Tell other things you are sure are true about that line.

For example

- You know that a line is of the form $y = mx + m$.
- What is true about that line?
- Would the same thing be true of lines of the form $y = mx + 2m$?

For example

- An equation is REALLY easy to solve.
- What could it be?

For example

- When working with linear relations, when would you find the table of values the best thing to have?
- When the graph?
- When the equation?

For example

- You have a table of values.
- It is REALLY EASY to figure out the equation that goes with it.
- What might the table look like?

For example

- When subtracting polynomials, when would it be easier NOT to add the opposite?

Your chance

- Try writing a “thinking” question on a topic you teach.

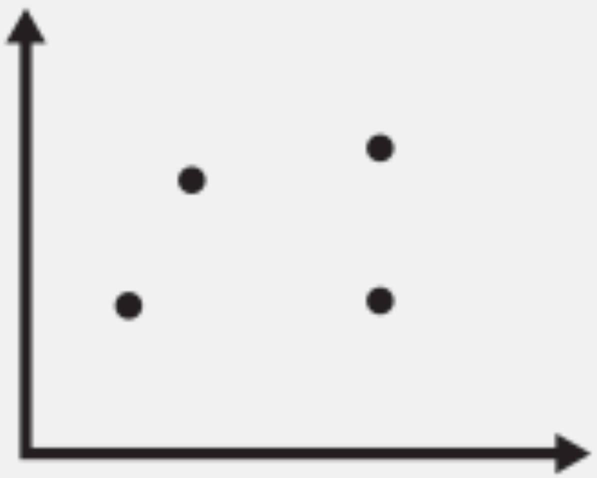
Open questions

- Lead to rich conversations
- Support differentiation by allowing access, but also stretching strong students

For example

A line passes through two of these points:

What could the equation
of the line be?



For example

- Choose values for m and b and graph

  $y = mx + b$.

For example

- The slope of a line is 8.
- One point on it is (4, 3).
- Name another point in Quadrant I.
- Name a third point in a different quadrant.

For example

- A right triangle has two really long sides and one short side.
- What could the side lengths be?

For example

- You need to use the Pythagorean theorem to figure out the area of a shape.
- Draw three such shapes that are different.
- Label dimensions and calculate the areas.

For example

- You saved $6 on an item at a sale.
- What might have been the original price?
- What was the percent discount for your original price?

For example

- 4L of juice that costs $8.28 is a **slightly** better buy than 3L at another price.
- What might the other price be and how do you know?

For example

- A system of equations is graphed.
- The solution is a point in Quadrant II.
- What could the equations be?

For example

- One acute angle in a quadrilateral is triple another angle. What could the four angles be?

Equations

- The solution to an equation is $x = -4$.
- What could the equation be?

Surface area and volume

- A and B are different kinds of shapes.
- A has a greater volume than B.
- B has a greater surface area than A.
- Sketch and show the dimensions.

Strategies

Begin with the answer, e.g.

- The volume of a cylinder is about 300 cm^3. What might its dimensions be?
- The difference of two polynomials is $-x^2 - 2x + 3$. What could they be?

Strategies

Ask for similarities and differences.

- How is solving $3x + 2 = 18 + x$ alike and different from solving the equation

  $-3x - 2 = -20 + x$?

Strategies

- How is figuring out the volume of a pyramid like figuring out the volume of a cone? How is it different?

Strategies

Leave out values

- E.g. _____% of ______ is 30.
- Fill in the blanks to make this true.

Strategies

- Choose a height for a cone.
- Choose a radius to make the cone narrow.
- Calculate the volume.

Strategies

Use “soft words”

- E.g.A line is MUCH steeper than and a LOT lower than $y = -x + 7$ when x is near 20.
- What might the equation be?

You can always start

- with a standard problem and change it.
- For example: What is the slope of the line that connects (4,3) and (2,1) becomes: A line has a slope of –1. What could the equation be?
- Or… A line with a slope of –1 goes through a point whose coordinates add to 7, What could the equation be?

Your turn

Create some open questions now.

Recall the strategies

- Start with answer
- Leave out numbers
- similarities/differenc
- Soft words

Parallel tasks

- Two similar tasks, often one being more complex in detail only.
- Sometimes one is more open than the other.
- Similar enough to be discussed together.

Similar triangles

- Choice 1: One triangle is similar to another. The 8 cm side in one triangle becomes 12 cm in the other. What happened to the 10 cm and 4 cm sides?
- Choice 2: One triangle is similar to another. The original sides are 4 cm, 10 cm and 8 cm. One of the sides becomes 12 cm. What could have happened to the other sides?

Common questions

- Did all the sides have to get larger? Why or why not?
- How did you decide what the enlargement factor was?
- Was there only one possibility?

Parallel Lines

- Choice 1: Figure out the equation of the line that is parallel to $2x + 7y = 3$ and goes through (14,9).
- Choice 2: Figure out the equation of the line that is parallel to $y = -2x + 3$ and goes through (14,9).

Common questions

- Will you figure out the slope of your new line first or the intercept? Why?
- What is the slope of your new line? How do you know?
- Did you need to know the new line went through (14,9)? Why?

Your questions

- If we are using open ended questions, how do we address developing numeracy/basic skills?

Your questions

- How do you get students to think more deeply and communicate their thinking vs merely regurgitating what they've learned/ memorized from their homework?

Your questions

- When using open-ended problems, how do you keep students motivated to push their thinking (some students will do it the easiest way they can when you know they are capable of more, some students just want to be "spoon fed")

Your questions

- How do you consolidate with kids after an open question?

Your questions

- How do we prepare them for the EQAO, grade 10 etc.?

Download

- www.onetwoinfinity.ca
- Waterloo